STABILITY IN A LAMINAR GAS BOUNDARY LAYER WITH SIMULTANEOUS HEATING AND COOLING

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Considerable interest attaches to controlling the transition from a laminar boundary layer to a turbulent one [1]. Until recently, it was assumed that surface cooling increases the boundary-layer stability in a gas and extends the transition to the turbulent state, whereas heating always produced the opposite effect [2]. However, it has been shown [3, 5], that those conclusions apply only for constant surface temperature. If the temperature distribution is uneven, heating with a negative temperature gradient can raise the stability considerably and narrow the unstable-frequency range, while cooling with a positive gradient has the opposite effect. This anomalous effect from heating has been confirmed by experiment [6]. Well-known laminarization methods (sucking off the boundary layer and cooling the surface) can be replaced by a new method: uneven heating. Also, even when the anomalous effect is not observed, the temperature gradient always has a substantial effect on the stability characteristics [7].

Here we consider the effects of simultaneous heating and cooling on the stability, which has not been examined before, but for which a preliminary analysis and the numerical calculations below show that it is effective in raising the stability if the heating section precedes the cooling one and the temperature gradient at the surface is negative. That combined method is promising and more economical than ones previously described, particularly because the heating and cooling can be realized in a thermodynamic cycle such as in a refrigerator.

We examined the stability in a subsonic laminar boundary layer of gas on a flat plate, whose surface temperature is a given function of the longitudinal coordinate $\psi_w = 1 + A(a + \xi^n)$ ($\xi = x/L$, L plate length, $\psi_w = T_w/T_e$, with the subscript w corresponding to the surface and e to the external flow). We derived the spectrum for the linear boundary-value problem

$$(L_1 + L_2)\varphi = 0;\ \varphi = 0,\ d\varphi/d\eta = 0,\ \eta = 0;\quad \varphi = 0,\ d\varphi/d\eta = 0,\ \eta = \infty. \tag{1}$$

Here $\varphi(\eta)$ is the current-function amplitude, L_1 and L_2 are differential operators [3, 5], with the differentiation with respect to the dimensionless coordinate $\eta = \left(\frac{u_e}{x\nu_e}\right)^{1/2}\int_0^y \frac{\rho}{\rho_e}dy$; u_e flow speed, ρ density, and ν kinematic viscosity. L_1 has the general form of an Orr–Sommerfeld operator, but with the difference that the term series includes the current temperature factor and the dimensionless viscosity. L_2 has a more cumbersome form and contains coefficients as expressions that themselves contain the distributions for the velocity, temperature, and gas physical properties. To derive the coefficients, it is necessary to solve the non-self-similar stationary temperature boundary layer problem subject to corresponding boundary conditions [5]. Keller's method is used to solve the equations numerically, with (1) solved by orthogonalization.

Figure 1 shows the calculations as curves for the neutral stability in the plane of the parameters: the dimensionless frequency $F = \omega\nu_e/u_e^2$, and the modified Reynolds number $R = (u_e x/\nu_e)^{1/2}$, together with the coefficients in the spatial amplification of inherent perturbations $-\alpha_i$ for fixed $R = 7\cdot10^4$. Curves 1 and 3 correspond to nonuniform heating and cooling with $a = -1$ and $n = 1$ for various heating–cooling depths: $A = -0.5$ (1); -0.2 (3), while curves 2 correspond to uneven cooling for $A = -0.5$, $a = 0$, $n = 1$; the dashed lines

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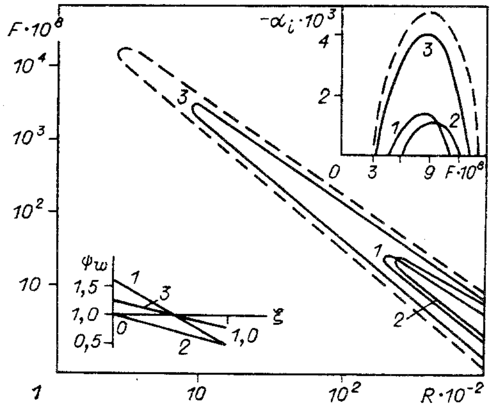

Fig. 1

represent isothermal conditions. For clarity, the bottom left corner shows the surface temperature distributions. The numbering corresponds to that used for the stability characteristics.

Comparison of curves 1 and 3 with the isothermal case shows that simultaneous heating and cooling raises the boundary-layer stability considerably, narrows the unstable-frequency range, and reduces the perturbation amplification coefficients. For A = −0.5 (curves 1), the critical $R_{min} = 2.1 \cdot 10^4$, while the isothermal value is $R_{min} = 300$. This corresponds to increase in the Re_{min} calculated from the distance from the leading edge of the plate by almost four orders of magnitude. The perturbation amplification coefficients are also much reduced. With this R, the $-\alpha_{i\,max}$ is reduced by more than a factor of three. With less vigorous heating and cooling, A = −0.2 (curve 2), the stabilization is naturally not so great, but still quite substantial (Re_{min} is increased by about a factor of eight).

It is of interest to compare the first case with the third, which corresponds to cooling the entire surface for x = L. The stability characteristics are almost identical.

Simultaneous heating and cooling is an effective way of stabilizing a laminar boundary layer. The effect is very much dependent on the extent of the heating and cooling, as is evident from the results for the first and second case.

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